

Confidential



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P1

MAY/JUNE 2024

MATHEMATICS: Paper 1



10611E

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages and 1 information sheet.

X05



INSTRUCTIONS AND INFORMATION

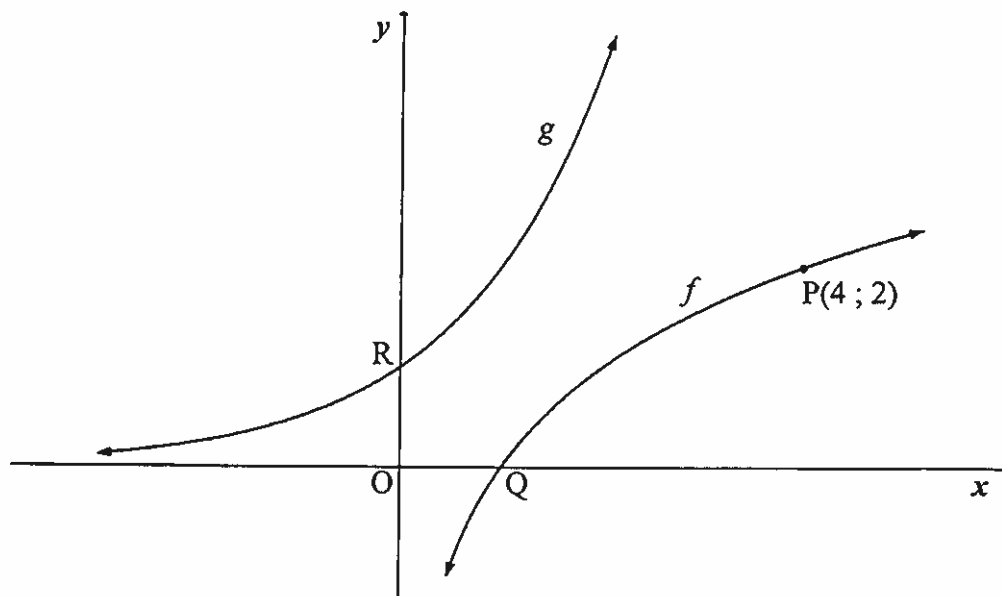
Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly



QUESTION 5

In the diagram, the graphs of $f(x) = \log_a x$ and g are drawn. Graph g is the reflection of f in the line $y = x$. Graph f passes through the point $P(4; 2)$. Q is the x -intercept of f and R is the y -intercept of g .

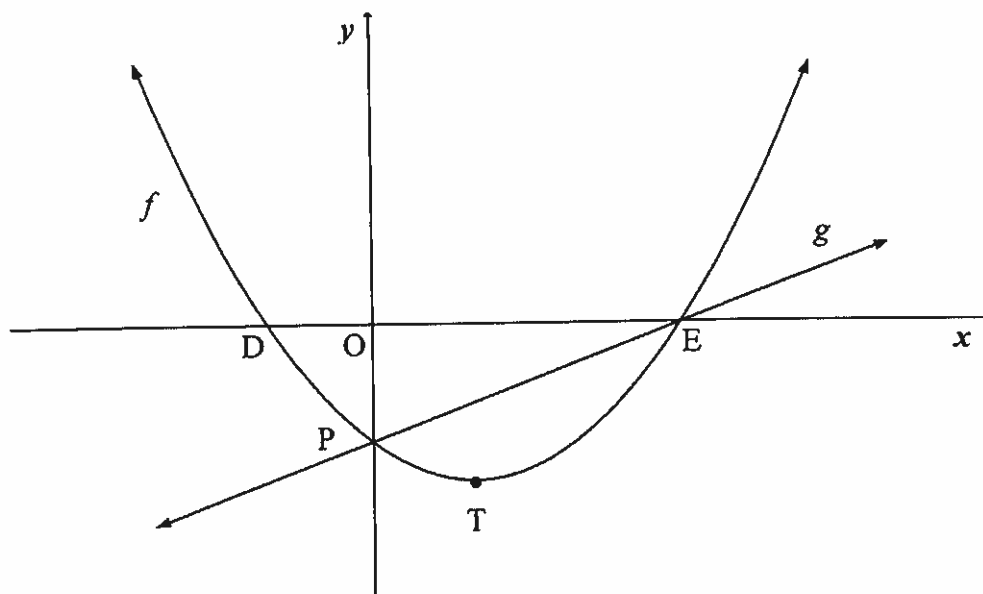


- 5.1 Write down the coordinates of P' , the image of P on g . (2)
- 5.2 Show that $a = 2$. (2)
- 5.3 Write down the equation of g in the form $y = \dots$ (1)
- 5.4 T is a point on f in the first quadrant where TR is parallel to the x -axis. Calculate the area of $\triangle RTP'$. (4)
- [9]



QUESTION 6

The graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = mx + c$ are drawn below. D and E are the x -intercepts and P is the y -intercept of f . The turning point of f is $T(1; -4)$. The graphs of f and g intersect at P and E.



- 6.1 Write down the range of f . (1)
- 6.2 Calculate the coordinates of D and E. (3)
- 6.3 Determine the equation of g . (2)
- 6.4 Write down the values of x for which $f(x) - g(x) > 0$. (2)
- 6.5 Determine the maximum vertical distance between h and g if $h(x) = -f(x)$ for $x \in [-2; 3]$. (5)
- 6.6 Given: $k(x) = g(x) - n$. (5)
- Determine n if k is a tangent to f . [18]



QUESTION 7

7.1 Six years ago, Thabo bought a phone for R13 000. The value of the phone depreciated annually according to the reducing-balance method. The value of the phone is now R8 337,75. Calculate the annual rate of depreciation. (3)

7.2 Eric and Thandi need to save R80 000 each to go on a holiday at the end of December 2027.

- Thandi decides that she will start saving at the end of January 2025. She will make 36 monthly deposits into a savings account that pays interest at 8,6% p.a., compounded monthly. The deposit will be made at the end of each month.
- Eric calculates that if he makes 48 deposits of R1 402,31, starting at the end of January 2024, he will have enough money to go on holiday. He will make his deposits into a savings account at the end of each month. The savings account pays interest at 8,6% p.a., compounded monthly.

Calculate the difference between the total amount that Eric and Thandi will deposit into their respective savings accounts over the given period. (4)

7.3 Lesibana was granted a loan of R225 000. The rate of interest for the loan is 9% p.a., compounded monthly. Lesibana will make monthly payments of R5 500, starting exactly four months after the loan was granted. How many payments will Lesibana make to settle the loan? (6)
[13]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = \frac{1}{x}$. (5)

8.2 Determine:

8.2.1 $\frac{d}{dx}(\sqrt{4x^6} + \sqrt{2} \cdot x^2)$ (3)

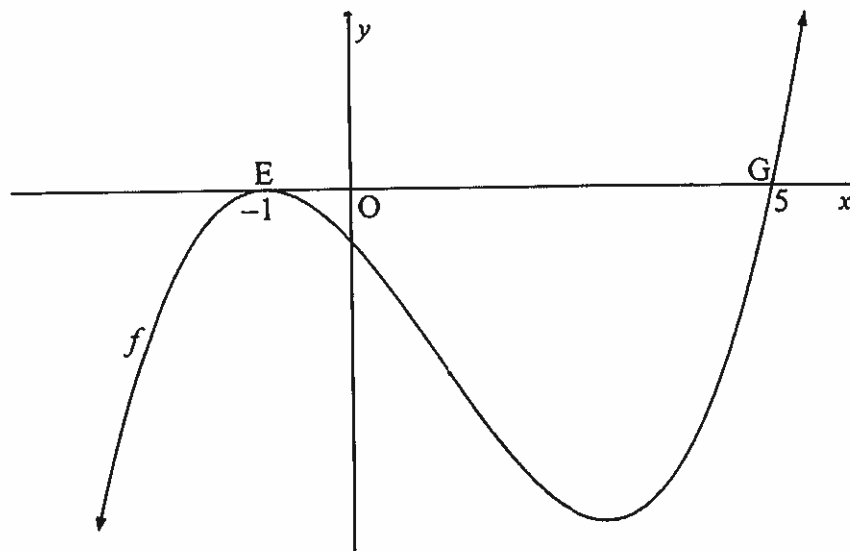
8.2.2 $g'(x)$ if $g(x) = \frac{3x^4 - 4x^2 + 6}{x^2}$ (3)

8.3 The equation of the tangent to $f(x) = 3x^2 + bx + c$ at $x = 1$ is given by $y = 9x - 9$. Determine the values of b and c . (4)
[15]



QUESTION 9

The graph of $f(x) = ax^3 + bx^2 + cx - 5$ is drawn below. E(-1 ; 0) and G(5 ; 0) are the x -intercepts of f .

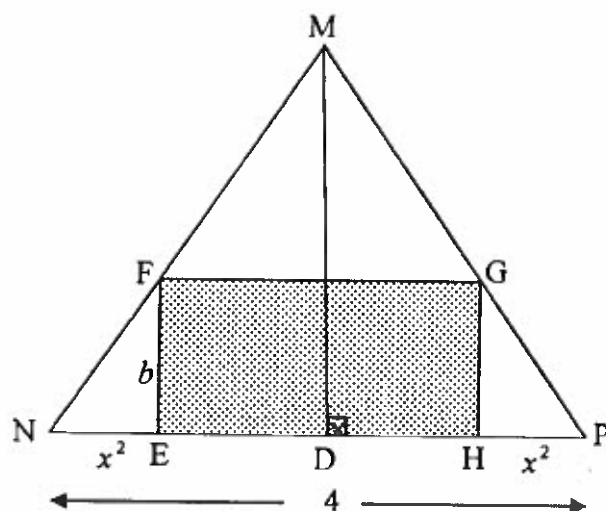


- 9.1 Show that $a = 1$, $b = -3$ and $c = -9$. (3)
- 9.2 Calculate the value of x for which f has a local minimum value. (4)
- 9.3 Use the graph to determine the values of x for which $f''(x) \cdot f(x) > 0$. (3)
- 9.4 For which values of t will the graph of $p(x) = f(x) + t$ have two distinct positive roots and one negative root? (3)
- [13]



QUESTION 10

EHGF is a rectangle. HE is produced x^2 cm to N and EH is produced x^2 cm to P. NF produced intersects PG produced at M to form an isosceles triangle MNP with $NM = MP$. D lies on NP where $MD \perp NP$. $NP = 4$ cm and $MD = 3$ cm.



10.1 Show that the area of EFGH is given by $A(x) = 6x^2 - 3x^4$. (4)

10.2 Calculate the maximum area of rectangle EFGH. (4)
[8]



QUESTION 11

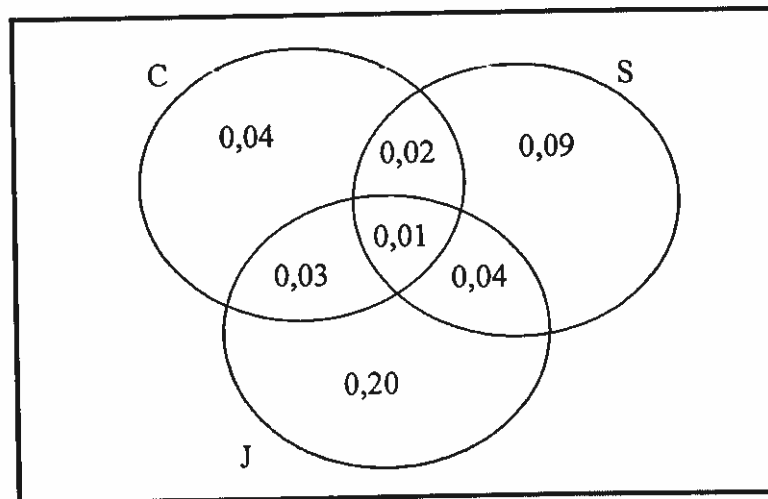
11.1 Two events, A and B, are such that:

- $P(A) = 0,4$
- $P(A \text{ or } B) = 0,52$
- A and B are mutually exclusive

Calculate $P(B)$.

(2)

11.2 The items that a learner bought at a tuck shop were recorded over a period of time. The probabilities of the learner buying a sandwich (S), a chocolate (C) and a juice (J) are shown in the Venn diagram below.



11.2.1 What is the probability that the learner will buy a sandwich?

(1)

11.2.2 Calculate the probability that the learner will buy at least two of the three items.

(2)

11.2.3 Calculate the probability that the learner would NOT buy any of the three items.

(2)



- 11.3 Seven guitar players, each with a different name, participate in a concert.
- 11.3.1 In how many different ways can the names of the guitar players be listed, one below the other, in the programme? (1)
- 11.3.2 After the performance, the guitar players wait backstage. There is a bench with only room for four to sit on.
- What will be the probability that the four guitar players will be sitting in alphabetical order, from left to right? (3)
- 11.3.3 During the performance, the seven guitar players sit in a line on stage. Four guitar players are female and three are male.
- In how many different ways can they be seated if the males may not sit next to each other? (3)
- [14]
- TOTAL: 150**



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$





basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIORSERTIKAAT-EKSAMEN/ NASIONALE SENIORSERTIFIKAAT-EKSAMEN

WISKUNDE V1

MEI/JUNIE 2024

WISKUNDE: Vraestel 1

PUNTE: 150

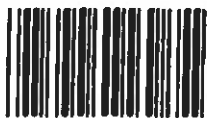


10611A

TYD: 3 uur

X05

Hierdie vraestel bestaan uit 11 bladsye en 1 inligtingsblad.



INSTRUKSIES EN INLIGTING

Lees die volgende instruksies noukeurig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit 11 vrae.
2. Beantwoord AL die vrae.
3. Nommer die antwoorde korrek volgens die nommeringstelsel wat in hierdie vraestel gebruik is.
4. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy gebruik het om jou antwoorde te bepaal, duidelik aan.
5. Volpunte sal NIE noodwendig aan slegs antwoorde toegeken word NIE.
6. Jy mag 'n goedgekeurde, wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.
7. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
8. Diagramme is NIE noodwendig volgens skaal geteken NIE.
9. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
10. Skryf netjies en leesbaar.



QUESTION 11.1 Solve for x :

1.1.1 $3x^2 + 5x = 0$ (2)

1.1.2 $4x^2 + 3x - 5 = 0$ (answers correct to TWO decimal places) (3)

1.1.3 $(x-1)^2 - 9 \geq 0$ (4)

1.1.4 $5^{2x} - 5^x = 0$ (4)

1.1.5 $\frac{x}{\sqrt{20-x}} = 1$ (5)

1.2 Solve for x and y simultaneously:

$x + y = 9$ and $2x^2 - y^2 = 7$ (5)

1.3 Given: $P = (1-a)$ and $T = (1+a)(1+a^2)(1+a^4) \dots (1+a^{512})$ Determine the value of $P \times T$ in terms of a . (3)
[26]**QUESTION 2**2.1 Consider the geometric series: $4 + 2 + 1 + \frac{1}{2} + \dots$

2.1.1 Does this series converge? Justify your answer. (2)

2.1.2 Calculate S_{∞} . (2)2.2 Given: $\sum_{p=k}^{10} 3^{p-1} = 29\,520$. Calculate the value of k .(5)
[9]

QUESTION 3

3.1 Consider the quadratic number pattern: 3 ; 7 ; 12 ; ...

3.1.1 Show that the general term of this number pattern is given by

$$T_n = \frac{1}{2}n^2 + \frac{5}{2}n. \quad (3)$$

3.1.2 What number must be added to T_{n-1} so that $T_n = 13\,527$? (4)

3.2 Given an arithmetic sequence with $T_1 = 8$ and $T_2 = 11$.

3.2.1 Calculate the value of n if $T_n = 41$. (3)

3.2.2 A new arithmetic sequence P is formed using the term position and the term value of the given arithmetic sequence.

For the new sequence, $P_8 = 1$, $P_{11} = 2$ and so forth.

(a) Write down the value of P_{41} . (1)

(b) Calculate the value of the first term of the new arithmetic sequence. (4)
[15]

QUESTION 4

Given: $g(x) = \frac{1}{x-1} + 2$

4.1 Write down the equations of the asymptotes of g . (2)

4.2 Draw a graph of g , indicating any intercepts with the axes and asymptotes. (4)

4.3 Determine the values of x where $g(x) > 0$. (2)

4.4 Determine the equation of the axis of symmetry of g which has a negative gradient. (2)
[10]



VRAAG 11.1 Los op vir x :

1.1.1 $3x^2 + 5x = 0$ (2)

1.1.2 $4x^2 + 3x - 5 = 0$ (antwoorde korrek tot TWEE desimale plekke) (3)

1.1.3 $(x-1)^2 - 9 \geq 0$ (4)

1.1.4 $5^{2x} - 5^x = 0$ (4)

1.1.5 $\frac{x}{\sqrt{20-x}} = 1$ (5)

1.2 Los gelyktydig vir x en y op:

$x + y = 9$ en $2x^2 - y^2 = 7$ (5)

1.3 Gegee: $P = (1-a)$ en $T = (1+a)(1+a^2)(1+a^4) \dots (1+a^{512})$ Bepaal die waarde van $P \times T$ in terme van a . (3)
[26]**VRAAG 2**2.1 Beskou die meetkundige reeks: $4 + 2 + 1 + \frac{1}{2} + \dots$

2.1.1 Konvergeer die reeks? Motiveer jou antwoord. (2)

2.1.2 Bereken S_{∞} . (2)2.2 Gegee: $\sum_{p=k}^{10} 3^{p-1} = 29\,520$. Bereken die waarde van k . (5)
[9]

VRAAG 3

3.1 Beskou die kwadratiese getalpatroon: 3 ; 7 ; 12 ; ...

3.1.1 Dui aan dat die algemene term van hierdie getalpatroon gegee word deur

$$T_n = \frac{1}{2}n^2 + \frac{5}{2}n. \quad (3)$$

3.1.2 Watter getal moet by T_{n-1} getel word sodat $T_n = 13\,527$? (4)

3.2 Gegee 'n rekenkundige reeks met $T_1 = 8$ en $T_2 = 11$.

3.2.1 Bereken die waarde van n indien $T_n = 41$. (3)

3.2.2 'n Nuwe rekenkundige reeks P word gevorm deur die termposisie en die termwaarde van die gegewe rekenkundige reeks te gebruik.
 $P_8 = 1$, $P_{11} = 2$ vir die nuwe reeks, en so aan.

(a) Skryf die waarde van P_{41} neer. (1)

(b) Bereken die waarde van die eerste term van die nuwe rekenkundige reeks. (4)
[15]

VRAAG 4

Gegee: $g(x) = \frac{1}{x-1} + 2$

4.1 Skryf die vergelykings van die asimptote van g neer. (2)

4.2 Teken 'n grafiek van g en dui enige afsnitte met die asse en asimptote aan. (4)

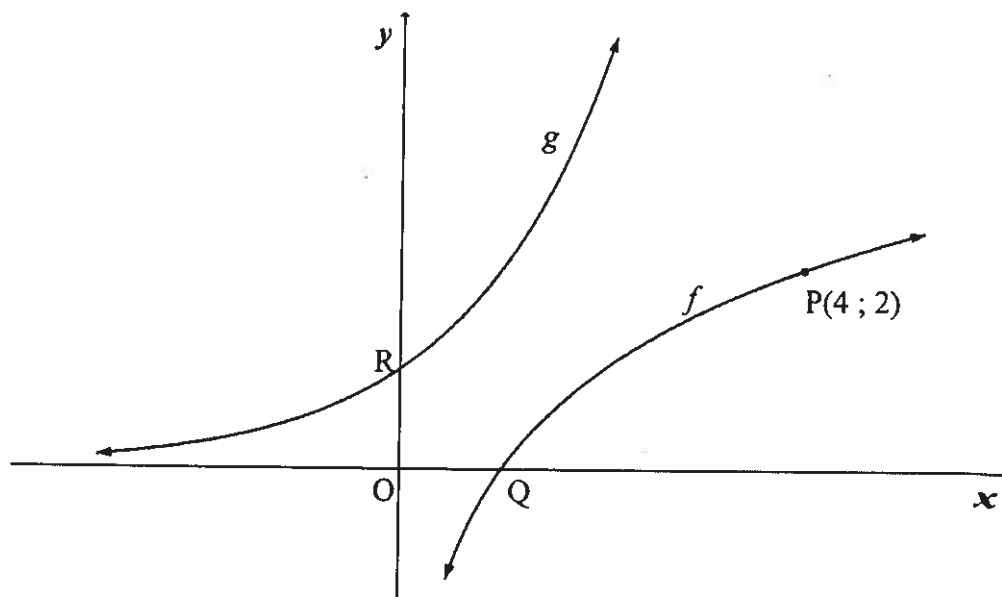
4.3 Bepaal die waardes van x waarvoor $g(x) > 0$. (2)

4.4 Bepaal die vergelyking van die simmetrie-as van g wat 'n negatiewe gradiënt het. (2)
[10]



VRAAG 5

In die diagram is die grafieke van $f(x) = \log_a x$ en g geskets. Grafiek g is die refleksie van f in die lyn $y = x$. Grafiek f gaan deur die punt $P(4; 2)$. Q is die x -afsnit van f en R is die y -afsnit van g .

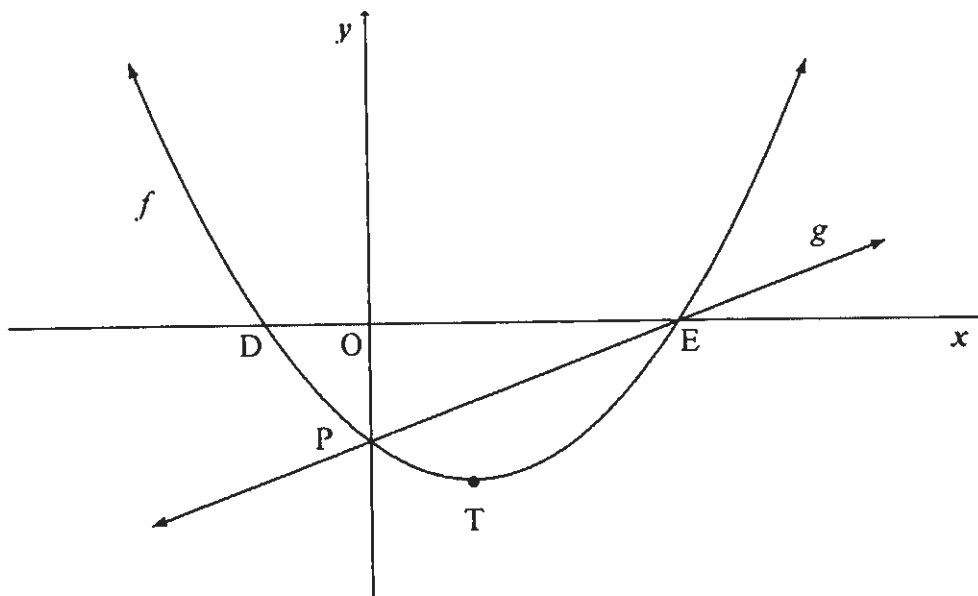


- 5.1 Skryf die koördinate van P' , die beeld van P op g , neer. (2)
- 5.2 Toon dat $a = 2$. (2)
- 5.3 Skryf die vergelyking van g in die vorm $y = \dots$ neer. (1)
- 5.4 T is 'n punt op f in die eerste kwadrant, met TR ewewydig aan die x -as. Bereken die oppervlakte van $\triangle RTP'$. (4)
- [9]



VRAAG 6

Die grafieke van $f(x) = x^2 - 2x - 3$ en $g(x) = mx + c$ is hieronder geskets. D en E is die x -afsnitte en P is die y -afsnit van f . Die draaipunt van f is $T(1; -4)$. Die grafieke van f en g sny mekaar by P en E.



- 6.1 Skryf die waardeversameling van f neer. (1)
- 6.2 Bereken die koördinate van D en E. (3)
- 6.3 Bepaal die vergelyking van g . (2)
- 6.4 Skryf die waardes van x neer waarvoor $f(x) - g(x) > 0$. (2)
- 6.5 Bepaal die maksimum vertikale afstand tussen h en g indien $h(x) = -f(x)$ vir $x \in [-2; 3]$. (5)
- 6.6 Gegee: $k(x) = g(x) - n$. (5)
- Bepaal n indien k 'n raaklyn aan f is. [18]



VRAAG 7

- 7.1 Thabo het ses jaar gelede 'n foon vir R13 000 gekoop. Die waarde van die foon het jaarliks volgens die verminderdesaldo-metode verminder. Die waarde van die foon is nou R8 337,75. Bereken die jaarlikse depresiasiekoers. (3)
- 7.2 Eric en Thandi moet elkeen R80 000 spaar om aan die einde van Desember 2027 met vakansie te gaan.
- Thandi besluit dat sy teen die einde van Januarie 2025 sal begin spaar. Sy sal 36 maandelikse deposito's in 'n spaarrekening maak, wat teen 8,6% p.j. rente betaal, maandeliks saamgestel. Die deposito sal aan die einde van elke maand gemaak word.
 - Eric bereken dat as hy aan die einde van Januarie 2024 begin en 48 deposito's van R1 402,31 maak, hy genoeg geld sal hê om met vakansie te gaan. Hy sal sy deposito's aan die einde van elke maand in 'n spaarrekening maak. Die spaarrekening betaal rente teen 8,6% p.j., maandeliks saamgestel.
- Bereken die verskil tussen die totale bedrag geld wat Eric en Thandi oor die gegewe tydperk in hulle onderskeie spaarrekenings sal deponeer. (4)
- 7.3 'n Lening van R225 000 is aan Lesibana toegestaan. Die rentekoers van die lening is 9% p.j., maandeliks saamgestel. Lesibana sal maandelikse betalings van R5 500 maak, met die eerste betaling presies vier maande nadat die lening toegestaan is. Hoeveel betalings sal Lesibana maak om die lening te vereffen? (6)
[13]

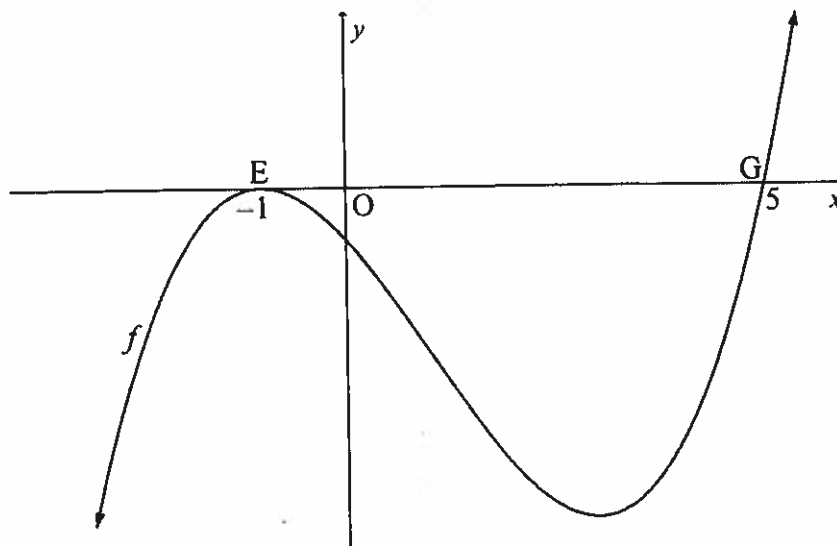
VRAAG 8

- 8.1 Bepaal $f'(x)$ vanuit eerste beginsels indien $f(x) = \frac{1}{x}$. (5)
- 8.2 Bepaal:
- 8.2.1 $\frac{d}{dx}(\sqrt{4x^6} + \sqrt{2} \cdot x^2)$ (3)
- 8.2.2 $g'(x)$ indien $g(x) = \frac{3x^4 - 4x^2 + 6}{x^2}$ (3)
- 8.3 Die vergelyking van die raaklyn aan $f(x) = 3x^2 + bx + c$ by $x = 1$ word gegee deur $y = 9x - 9$. Bepaal die waardes van b en c . (4)
[15]



VRAAG 9

Die grafiek van $f(x) = ax^3 + bx^2 + cx - 5$ is hieronder geskets. $E(-1; 0)$ en $G(5; 0)$ is die x -afsnitte van f .

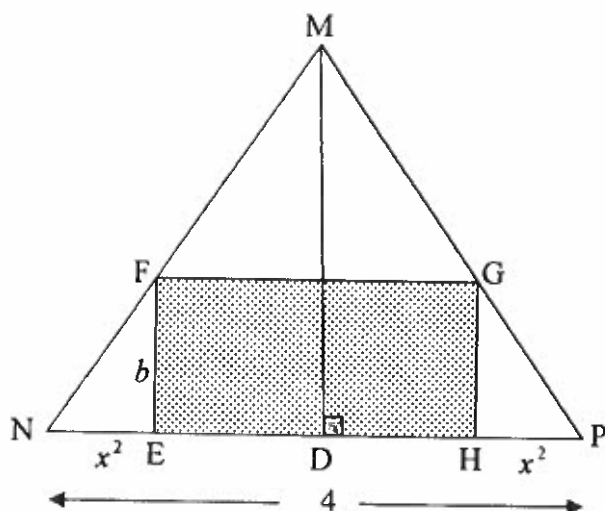


- 9.1 Toon dat $a = 1$, $b = -3$ en $c = -9$. (3)
- 9.2 Bereken die waarde van x waarvoor f 'n lokale minimum waarde het. (4)
- 9.3 Gebruik die grafiek om die waardes van x te bepaal waarvoor $f''(x) \cdot f(x) > 0$. (3)
- 9.4 Vir watter waardes van t sal die grafiek van $p(x) = f(x) + t$ twee verskillende positiewe wortels en een negatiewe wortel hê? (3)
- [13]



VRAAG 10

EHGF is 'n reghoek. HE word x^2 cm verleng na N en EH word x^2 cm verleng na P. NF verleng, sny PG verleng by M om 'n gelykbenige driehoek MNP met $NM = MP$ te vorm. D is 'n punt op NP waar $MD \perp NP$. $NP = 4$ cm en $MD = 3$ cm.



- 10.1 Toon dat die area van EFGH deur $A(x) = 6x^2 - 3x^4$ gegee word. (4)
- 10.2 Bereken die maksimum oppervlakte van reghoek EFGH. (4)
- [8]



VRAAG 11

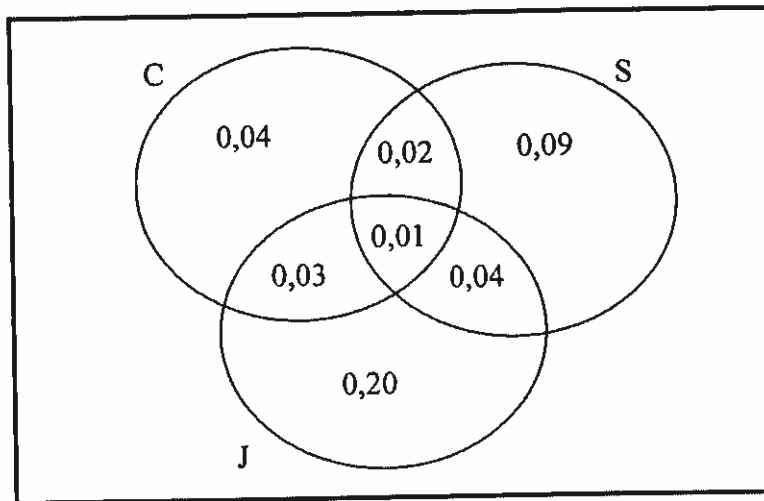
11.1 Twee gebeurtenisse, A en B, is sodanig dat:

- $P(A) = 0,4$
- $P(A \text{ of } B) = 0,52$
- A en B is onderling uitsluitend

Bepaal $P(B)$.

(2)

11.2 Die items wat 'n leerder oor 'n sekere tydperk by 'n snoepwinkel gekoop het, is aangeteken. Die waarskynlikheid dat die leerder 'n toebroodjie (S), 'n sjokolade (C) en 'n vrugtesap (J) koop, word in die Venn-diagram hieronder voorgestel.



- 11.2.1 Wat is die waarskynlikheid dat die leerder 'n toebroodjie sal koop? (1)
- 11.2.2 Bereken die waarskynlikheid dat die leerder ten minste twee van die drie items sal koop. (2)
- 11.2.3 Bereken die waarskynlikheid dat die leerder NIE een van die drie items sal koop NIE. (2)



- 11.3 Sewe kitaarspelers, elkeen met 'n ander naam, neem aan 'n konsert deel.
- 11.3.1 Op hoeveel verskillende maniere kan die name van die kitaarspelers onder mekaar in die program gelys word? (1)
- 11.3.2 Na die vertoning wag die kitaarspelers agter die verhoog. Daar is 'n bankie met plek vir slegs vier om op te sit.
- Wat sal die waarskynlikheid wees dat die vier kitaarspelers in alfabetiese volgorde, van links na regs, sal sit? (3)
- 11.3.3 Gedurende die vertoning sit die sewe kitaarspelers in 'n ry op die verhoog. Vier kitaarspelers is vroulik en drie is manlik.
- Op hoeveel verskillende maniere kan die spelers sit indien die mans nie langs mekaar mag sit nie? (3)
- [14]
- TOTAAL: 150**



INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

