KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PRACTICE

JUNE 2024

MARKING GUIDELINE

MARKS: **150**

TIME: **3 HOURS**

QUESTION 1

1.1	$m_{AD} = m_{BC} = \frac{3+1}{2-6}$ $= -1$ $y = -x + c$ Sub A(-3 ; 3): $3 = -(-3) + c$ $c = 0$ $\therefore y = -x$	✓substitution ✓ m ✓substitution ✓ equation (4)
1.2	$D(x\ ;\ y) \rightarrow y = -x$ BD = CD → BD² = CD²: $(2-x)^2 + (3-y)^2 = (6-x)^2 + (-1-y)^2$ $(2-x)^2 + \left(3-(-x)\right)^2 = (6-x)^2 + \left(-1-(-x)\right)^2$ $4 - 4x + x^2 + 9 + 6x + x^2 = 36 - 12x + x^2 + 1 - 2x + x^2$ $16x = 24$ $x = \frac{24}{16} = \frac{3}{2}$ $y = -\frac{3}{2}$ $\therefore D\left(\frac{3}{2}\ ;\ -\frac{3}{2}\right)$	✓ BD = CD ✓substitution ✓ $y = -x$ ✓simplification ✓ x -value ✓ y -value (6)
1.3	$m_{BC} = \frac{3+\frac{3}{2}}{2-\frac{3}{2}} = 9$	✓substitution ✓ answer (2)
1.4	let ∠ of inclination of BC be α and ∠ of inclination of BD be β $\tan\alpha = m_{BC}$; $\tan\beta = m_{BD}$ $\tan\alpha = -1$; $\tan\beta = 9$ $\alpha = 135°$; $\beta = 83{,}7°$ $\theta = \alpha - \beta$ $= 135° - 83{,}7°$ $= 51{,}3°$	✓ tan = m ✓ angles ✓difference ✓answer (4)
1.5	$BD^2 = CD^2 = \left(2-\left(\frac{3}{2}\right)\right)^2 + \left(3-\left(-\frac{3}{2}\right)\right)^2 = \frac{41}{2}$ $\therefore BD = CD = \frac{\sqrt{82}}{2}$ units $B\hat{D}C = 180° - (51{,}3° \times 2) = 77{,}4°$ $\text{Area}_{\Delta ABC} = \frac{1}{2}\left(\frac{41}{2}\right)\sin 77{,}4° = 10$ square units	✓substitution ✓BD=CD ✓ $B\hat{D}C$ ✓sub in Area Rule ✓answer (5)

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QUESTION 2

2.1.1	$x^2+8x+(4)^2+y^2-6y+(-3)^2 = -9+(4)^2+(-3)^2$ $(x+4)^2+(y-3)^2 = 16$ $\therefore$ centre $(-4\,;3)$	✓substitution ✓ centre-radius form ✓answer (3)
2.1.2	Circle O: $r_O = 1$ unit O (-1 ; 3) Circle M: $r_M = 4$ units OM = 3 units Since: OM = $r_M - r_O$ $\therefore$ circles touch internally	✓ r_O ✓r_M ✓ OM ✓conclusion (4)
2.2.1	$y = x+2$ $\therefore m_{AC} = 1$ $\Rightarrow m_{AB} = -1$ tan $\perp$ rad sub M(4 ; 4) $4 = -(4) + c$ $c = 8$ $\therefore y = -x+8$	 ✓ S/R ✓substitution ✓eqn (3)
2.2.2	At A: $x+2 = -x+8$ $2x = 6$ $x = 3$ $y = (3)+2 = 5$ $\therefore$ A (3 ; 5)	✓equating ✓ x-val (2)
2.2.3	$(x-4)^2+(y-4)^2 = r^2$ sub A (3 ; 5): $r^2 = (3-4)^2+(5-4)^2 = 2$ $\therefore (x-4)^2+(y-4)^2 = 2$	✓$(x-4)$ $(y-4)$ ✓sub ✓r ✓eqn (4)
2.2.4	$\frac{x_B+3}{2} = 4$ $\frac{y_B+5}{2} = 4$ $x_B + 3 = 8$ $y_B + 5 = 8$ $x_B = 5$ $y_B = 3$ $\therefore$ B(5 ; 3)	 ✓ x_B ✓ y_B Answer Only: full marks (2)
2.2.5	AC || BD (co-int ∠s =) $m_{AC} = m_{BD} = 1$ sub B(5 ; 3): $3 = 5 + c$ $c = -2$ $\therefore y = x-2$	✓ S/R ✓ = m ✓substitution ✓eqn (4)

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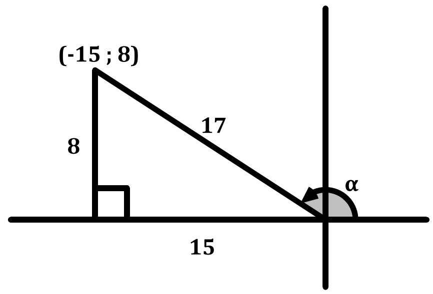
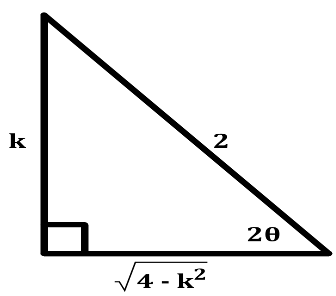
QUESTION 3

3.1	$\sin\alpha = \frac{8}{17}$ $x = \sqrt{(17)^2 - (8)^2}$ $x = -15$ 	✓diagram
3.1.1	$= 3\left(\frac{8}{-15}\right) = -\frac{8}{5}$	✓x-val ✓answer (3)
3.1.2	$= \cos\alpha$ $= -\frac{15}{17}$	✓$\cos\alpha$ ✓answer (2)
3.1.3	$= \cos^2\alpha - \sin^2\alpha$ $= \left(-\frac{15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$	✓expansion ✓substitution ✓answer (3)
3.2	$\sin\theta\cos\theta = \frac{k}{4}$ $(\times 2): 2\sin\theta\cos\theta = \frac{k}{2}$ $\sin 2\theta = \frac{k}{2}$ $\therefore \tan 2\theta = \frac{k}{\sqrt{4-k^2}}$ 	✓$\times 2$ ✓$\sin 2\theta$ ✓diagram ✓ x-val / adj ✓answer (5)

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QUESTION 4

4.1	$= \dfrac{2\cos(90° + 15°)\cos 15°}{\cos(45° - x + x)}$ $= \dfrac{2\sin 15° \cos 15°}{\cos 45°}$ $= \dfrac{\sin 30°}{\cos 45°} = \dfrac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}$ $= \dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$	✓$\cos(45° - x + x)$ ✓ $\sin 15°$ ✓ $\cos 45°$ ✓ $\cos 30°$ ✓ $\dfrac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}$ ✓answer (6)
4.2.1	$\text{RHS} = \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} \times \dfrac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}$ $= \dfrac{(\cos\theta + \sin\theta)^2}{\cos^2\theta - \sin^2\theta}$ $= \dfrac{\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta}{\cos 2\theta}$ $= \dfrac{1 + \sin 2\theta}{\cos 2\theta} = \text{LHS}$ **OR**	✓ ×conjugate ✓$\cos^2\theta - \sin^2\theta$ ✓expansion ✓ $\cos 2\theta$ ✓ 1 (5)
	$\text{LHS} = \dfrac{\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$ $= \dfrac{(\cos\theta + \sin\theta)^2}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$ $= \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \text{RHS}$	✓expansion ✓$2\sin\theta\cos\theta$ ✓$\cos^2\theta - \sin^2\theta$ ✓square ✓factors (5)
4.2.2	$\cos 2\theta = 0 \quad ref\ \angle = 90°$ $2\theta = 90° + k.360°$ $\theta = 45° + k.180°\ ; k \in \mathbb{Z}$	✓ $\cos 2\theta = 0$ ✓ $45° + k.180°$ ✓ $k \in \mathbb{Z}$ (3)
4.2.3	$= \dfrac{1 + \sin 30°}{\cos 30}$ $= \dfrac{1 + \frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $= \sqrt{3}$	✓ ✓ substitution ✓answer (3)
4.3	$7\cos x - 2(1 - \cos^2 x) + 5 = 0$ $7\cos x - 2 + 2\cos^2 x + 5 = 0$ $2\cos^2 x + 7\cos x + 3 = 0$ $(2\cos x + 1)(\cos x + 3) = 0$ $\cos x = -\frac{1}{2}$ or $\underline{\cos x = -3}$ $x = 120° + k.360°$ $\therefore$ n/a $x = 240° + k.360°\ ; k \in \mathbb{Z}$	✓expansion ✓std form ✓factors ✓$\cos x = -\frac{1}{2}$ ✓$\cos x = -3$ ✓✓answers (7)

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QUESTION 5

5.1	a = 2 b = 2	✓ ✓ (2)
5.2	f: 180° g: 360°	✓ ✓ (2)
5.3	amplitude: 1	✓ (1)
5.4	$y \in [2\ ;\ 4]$	✓ end points ✓ notation (2)
5.5	$-180° < x < 0° \quad ; \quad x \neq -90°$ **OR**	✓end points ✓notation ✓ $x \neq -90°$ (3)
	$x \in (-180°\ ; 90°) \cup (90°\ ; 0°)$	✓✓end points ✓notation (3)
5.6	$h(x) = \sin(2x + 45°)$	✓ ✓ (2)
5.7	$k(x) = -2\cos x$	✓ ✓ (2)

[14]

QUESTION 6

6.1	$\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2 - 2.\mathrm{AB}.\mathrm{BC}.\cos \mathrm{A\hat{B}C}$ $= a^2 + 4a^2 - 2(a)(2a).\cos 2\beta$ $\therefore \mathrm{AC} = \sqrt{5a^2 - 4a^2\cos 2\beta}$ $= \sqrt{a^2(5 - 4\cos 2\beta)}$ $= a\sqrt{5 - 4\cos 2\beta}$	✓cosine rule ✓substitution ✓answer (3)
6.2	In ΔADC: $\tan\beta = \dfrac{\mathrm{AD}}{a\sqrt{5 - 4\cos 2\beta}}$ $\mathrm{AD} = \tan\beta \cdot a\sqrt{5 - 4\cos 2\beta}$ $= a\tan\beta\sqrt{5 - 4(1 - 2\sin^2\beta)}$ $= a\tan\beta\sqrt{5 - 4 + 8\sin^2\beta}$ $= a\tan\beta\sqrt{1 + 8\sin^2\beta}$	✓ trig ratio ✓substitution ✓$1 - 2\sin^2\beta$ ✓simplification (4)

[7]

QUESTION 7

7.1.1	$100°$	ext ∠ of Δ	✓S ✓R (2)
7.1.2	$50°$	∠ at cent = 2∠ at CFCE	✓S ✓R (2)
7.1.3	$130°$	opp ∠s of cyclic quad = 180°	✓S ✓R (2)
7.1.4	$78° - 50° = 28°$	corres ∠s ; AOF || EH	✓S ✓R (2)
7.2	Let $\hat{C} = x$ $\therefore\ A\hat{D}B = x$ $\&\ A\hat{O}B = 2x$ $\hat{A}_1 = A\hat{D}B = x$ $\therefore\ \hat{E}_1 = 180° - 2x$ $\therefore\ \hat{E}_2 = 2x$ $\hat{E}_2 = A\hat{O}B = 2x$ $\Rightarrow$ AEOB is a cyclic quadrilateral	 ∠s in same segment ∠ at cent = 2∠ at CFCE alt ∠s; AC || BD sum of ∠s in Δ ∠s on a str. line converse ∠s in same segment	 ✓S/R ✓S/R ✓S/R ✓S/R ✓S/R ✓R (6)

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QUESTION 8

8.1	Construction: Draw KS = PQ and KT = PR. Join ST In ΔKST and ΔPQR: 1. KS = PQ (constr) 2. $\hat{K} = \hat{P}$ (given) 3. KT = PR (constr) $\therefore$ ΔKST ≡ ΔPQR (S; A; S) $\Rightarrow \hat{S}_1 = \hat{Q}$ (ΔKST ≡ ΔPQR) $\&\ \hat{Q} = \hat{L}$ $\therefore\ \hat{S}_1 = \hat{L}$ $\therefore$ ST || LM (corres ∠s =) $\therefore\ \frac{KL}{KS} = \frac{KM}{KT}$ (Prop. Int. Theorem; ST || LM) but KS = PQ & KT = PR $\therefore\ \frac{KL}{PQ} = \frac{KM}{PR}$	✓construction ✓S/R ✓S/R ✓R ✓S/R ✓S (6)

8.2.1	$\hat{F}$ $\hat{D}_1$	∠s opp = sides = tan-chord theorem	✓S ✓R ✓S ✓R (4)
8.2.2	$\hat{F} = \hat{D}_1$ $\therefore$ DE = EF	above sides opp = ∠s =	✓S ✓R (2)
8.2.3	$\hat{G}_2 = 180° - 2x$ $\therefore \hat{G}_1 = 2x$ $\therefore D\hat{O}E = 4x$	sum of ∠s in Δ ∠s on a str. line ∠ at cent = 2∠ at CFCE	✓S ✓R ✓S/R ✓S/R (4)
8.2.4	In ΔFDE and ΔFEG: 1. $\hat{F}$ is common 2. $\hat{D}_1 = \hat{E}_3$ $\therefore$ ΔFDE ||| ΔFEG $\therefore \frac{FD}{FE} = \frac{FE}{FG}$ $\Rightarrow FE^2 = FD \times FG$	 above (∠; ∠; ∠) ΔFDE ||| ΔFEG	 ✓S ✓S ✓R ✓S (4)

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QUESTION 9

9.1	$R\hat{V}W = 90°$ $R\hat{P}N = 90°$ $\therefore$ TW || SN	tan ⊥ radius line from cent to midpt of ch ⊥ ch corres ∠s =	✓S ✓R ✓S/R ✓R (4)
9.2	$\frac{RS}{ST} = \frac{RN}{NW}$ $\frac{10}{7} = \frac{RN}{6}$ $RN = \frac{60}{7}$ NK = RK – RN $= 10 - \frac{60}{7}$ $= \frac{10}{7}$ units	(Prop. Int. Theorem; SN || TW)	✓S ✓R ✓ RN ✓ ✓ (5)

<table>
<tr><td rowspan="3">9.3</td><td>$WV^2 = RW^2 - RV^2$ (Pythag)
$= \left(\frac{60}{7}+6\right)^2 - (10)^2$

WV = 10,598211

$\frac{PN}{WV} = \frac{RN}{RW}$ ($\Delta RPN \mid\mid\mid \Delta RVW$)

$\frac{PN}{10,598211} = \frac{\frac{60}{7}}{\frac{60}{7}+6}$

PN = 6,23 units</td><td>✓R
✓substitution
✓WV
✓S/R
✓substitution
✓PN (6)</td></tr>
<tr><td colspan="2">OR</td></tr>
<tr><td>$\frac{RP}{RV} = \frac{RS}{RT}$ (Prop. Int. Th; TW || SN)

$\frac{RP}{10} = \frac{10}{17}$

$RP = \frac{100}{17}$

$PN^2 = RN^2 - RP^2$ (Pythag)
$= \left(\frac{60}{7}\right)^2 - \left(\frac{100}{17}\right)^2$

= 6,23 units</td><td>✓S ✓R

✓RP

✓R
✓substitution
✓PN
(6)</td></tr>
</table>

[15]

TOTAL MARKS: 150